

First Semester M.Sc. in Physics Examination, September 2016
MATHEMATICAL METHODS FOR PHYSICS

Time: 3 Hours

Max. Marks : 80

Instruction : Answer all questions.

1. a) Let V be the vector space of all (2×2) matrices over the real field \mathbb{R} . Show that W is not a subspace of V where :
 - i) W consists of all matrices with zero determinant
 - ii) W consists of all matrices A for which $A^2 = A$.10
- b) Prove that the trace of a matrix remains invariant under similarity transformation. 5

OR

2. a) Calculate the Fourier transform (inverse transform) of $F(k) = \frac{N}{\sqrt{2\alpha}} e^{-\left(\frac{k^2}{4\alpha}\right)}$. Sketch an appropriate diagram for $f(x)$ and $F(k)$. 10
- b) Check whether the (3×3) rotation matrix can be diagonalized. 5

3. a) Prove that the following is a tensor of rank 2 in two dimensional space :

$$\bar{A} = \begin{bmatrix} y^2 & -xy \\ -xy & x^2 \end{bmatrix}. \quad 10$$

- b) Show that $(-y, x)$ are the components of a tensor of rank 1 in two dimension. 5

OR

4. a) Determine the conjugate metric Tensor in Spherical coordinate system. 10
- b) Express grad, div, Curl in Cylindrical coordinate system. 5
5. a) Separate the Helmholtz equation $(\nabla^2 + k^2)\psi = 0$ in cylindrical coordinates and identify the resulting ordinary differential equations. 10
- b) Find out the singularity of $(1-x)^2 y'' - 2xy' + n(n+1)y = 0$ at $x = \pm 1$. 5

OR

MP 1.1

6. a) Solve the integral equation $\phi(x) = x + \frac{1}{2} \int_{-1}^{+1} (1-x)\phi(t) dt$ using the Neumann series method. 10
- b) Describe the method of transformation of a differential equation into an integral equation. 5
7. a) Solve the Laguerre differential equation $xy'' + (1-x)y' + \lambda y = 0$ by using Frobenius' method. 10
- b) State and prove the orthogonality relation for Legendre polynomials. 5
- OR**
8. a) Prove that $e^{\left(\frac{x}{2}\right)\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(x)t^n$. 10
- b) Obtain the relationship between gamma and beta functions. 5
9. Answer any four of the following : **(4x5=20)**
- a) Determine whether or not the following vectors in R^3 are linearly dependent.
- (1, 2, 1), (2, 1, 1), (7, 4, 1)
 - (1, 2, 3), (1, 3, 2), (2, 1, 5).
- b) Show that the eigenvalues of a Hermitian matrix are real.
- c) Calculate the convolution, $f*g$, of $f(x) = g(x) = e^{-|x|}$.
- d) Find the two dimensional components of Christoffel symbols of first kind in plane polar coordinates.
- e) Define regular and irregular singular points of a second order ordinary differential equation.
- f) Define the Wronskian determinant. Check whether the following solutions are linearly independent or not ; $e^x \cos x$, $e^x \sin x$.
- g) Prove the orthogonality of $L_n(X)$.
- h) Show that $\frac{d}{dx} \{x^n J_n(x)\} = x^n J_{n-1}(x)$.