

I Semester M.Sc. Degree Examination, May 2014
PHYSICS : Mathematical Methods for Physics

Time : 3 Hours

Max. Marks : 80

Instruction : Answer all questions.

1. a) Discuss the effect of change of basis and similarity transformation in detail.
 b) Define linear vector space. (10+5)

OR

2. a) State and prove Schur's theorem on transformation.
 b) Obtain the Fourier transform of the Dirac function $\delta(t-a)$. (10+5)
3. a) Prove that the following is a tensor of rank 2 in 2-dimensional space

$$\bar{A} = \begin{pmatrix} y^2 & -xy \\ -xy & x^2 \end{pmatrix}.$$

- b) Define sum, difference and inner product of tensors. (10+5)

OR

4. a) Define Christoffel symbol of II kind and obtain the Christoffel symbol of II kind in plane polar coordinates.
 b) Express grad, div and curl in arbitrary rectangular coordinate system. (10+5)

5. a) Solve the differential equation $\frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 + 2)y = 0$ by Frobenius' method.
 b) Define regular and irregular singular points of a second order ordinary differential equation. (10+5)

OR

6. a) Describe the method of transformation of a second order linear differential equation into an integral equation.
 b) Derive a Fredholm integral equation corresponding to $y''(x) - y'(x) = 0$ with $y(1) = 1$ and $y(-1) = 1$. (10+5)

P.T.O.



7. a) Prove that the generating function of $L_n(x)$ satisfies the equation

$$\frac{1}{1-t} e^{\left(-\frac{xt}{1-t}\right)} = \sum_{n=0}^{\infty} L_n(x)t^n.$$

- b) Prove that the recurrence relation of $P_n(x)$ is given by

$$nP_n(x) = (2n-1)x P_{n-1}(x) - (n-1) P_{n-2}(x) \quad (10+5)$$

OR

8. a) Obtain Bessel's functions from the Bessel equation given by

$$x^2y'' + xy' + (x^2 - n^2)y = 0 \text{ by Frobenius' method.}$$

- b) Obtain the relationship between gamma function and beta function. (10+5)

9. Answer any four of the following : (5×4=20)

- a) Prove that all vector spaces of the same dimension are isomorphic.
- b) Determine whether or not the following vectors are linearly dependent in \mathbb{R}^3 $(1, 2, 1), (2, 1, 1)$ and $(7, 4, 1)$.
- c) Define and explain the types of tensor of rank 1 and tensor of rank 2.
- d) Prove that if A_p is a tensor that its covariant derivative $A_{p,q}$ is also a tensor.
- e) Check the singularity of the equation $(1-x^2)y'' - 2xy' + l(l+1)y = 0$ at $x = +1$ and $x = -1$.
- f) Define the Wronskian determinant. Check whether the following solutions are linearly independent or not : $e^x \cos x, e^x \sin x$.
- g) Prove that $xL'_n(x) = nL_n(x) - nL_{n-1}(x)$.
- h) Show that $xJ'_n(x) = xJ_{n-1}(x) - J_n(x)$.