Sl. No. : 0030

MP 1.1

Total No. of Pages : 3

First Semester M.Sc. Degree Examination, July/August - 2019
(SLM Scheme)
PHYSICS (Course - I)
Mathematical Methods of Physics

Time : 3 Hours **Max. Marks : 80**

Instruction : *Answer all questions.*

1. a) Discuss the effect of change of basis and similarity transformation in detail. **[10]**

   b) Prove that all vector spaces of the same dimension are isomorphic. **[5]**

OR

2. a) Prove that the eigen values of Hermitian matrix are real. **[5]**

   b) Is the matrix $\begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ diagonalizable? If yes, find the diagonalized matrix. **[5]**

   c) Calculate the cosine Fourier transforms of $e^{-\alpha x}$, where α is a positive integer. **[5]**

3. a) Prove that $\bar{A} = \begin{bmatrix} y^2 & -xy \\ -xy & x^2 \end{bmatrix}$ is a tensor of rank 2 in 2-dimensional space. **[10]**

   b) $A_{\alpha}^{\beta\gamma}$ and B_{δ}^{θ} are two tensors. Show that their outer product is also a tensor. **[5]**

OR

4. a) What are Christoffel symbols of I kind? Show that Christoffel symbols of I kind do not transform like tensors. **[10]**

   b) Obtain an expression for gradient of a scalar φ in terms of an arbitrary curvilinear coordinate system. **[5]**

5. a) Obtain the solutions for Helmholtz equation in spherical coordinate system using separation of variables. **[10]**

b) Solve the equation : $x\frac{d^2y}{dx^2}+(1-x)\frac{dy}{dx}+\lambda y=0$ using Frobenius method. **[5]**

OR

6. a) Convert a simple harmonic oscillator differential equation into an integral equation. **[5]**

b) Solve the integral equation $\varphi(x)=x+\frac{1}{2}\int_{-1}^{+1}(t-x)\varphi(t)\,dt$ using Neumann series method. **[5]**

c) Obtain a general solution for a non-homogeneous integral equation using Hilbert-Schmidt theory. **[5]**

7. a) Obtain the Legendre polynomials from the following differential equation

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0$$ **[10]**

b) Prove the Orthogonality condition of Laguerre polynomials. **[5]**

OR

8. a) Obtain the solutions to the Bessel's differential equation :

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-n^2)=0$$ **[10]**

b) Obtain the relation between beta and gamma functions. **[5]**

9. Answer **any four** of the following : **[4 × 5 = 20]**

a) Prove that the trace of a matrix remains invariant under similarity transformation.

b) Obtain the Fourier transform of Dirac delta function.

c) Explain the significance of "Jacobian" in co-ordinate transformation.

d) Obtain the Christoffel symbols of II kind in plane polar co-ordiantes.

e) Check the singularity of the equation $\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+l(l+1)y=0$ at $x=+1$ *and* $x=-1$.

f) Transform the differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+f(x)=0$ into an integral equation.

g) Prove that $P'_{n+1}(x)-P'_{n-1}(x)=(2n+1)P_n(x)$.

h) Show that $\Gamma\left(\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2}$.