MP 1.2

First Semester M.Sc. in Physics Examination, May 2015
CLASSICAL MECHANICS

Time : 3 Hours | Max. Marks : 80

Instruction : *Answer* ***all*** *questions.*

1. a) Define center of mass for a system of particles. **5**

   b) Prove that the total angular momentum of a system of particles is conserved in the absence of applied torques. Obtain an expression for angular frequency of a charged particle moving in magnetic field. **10**

OR

2. a) With schematic diagrams, write down the equations of constraints in the case of a simple pendulum moving in the xy plane and a particle moving on or outside the surface of a sphere of radius a. **5**

   b) Starting from D'Alembert's principle obtain Lagrange's equations of motion for the conservative system. **10**

3. a) Solve the inverse Kepler problem using Binet's equation and show that the central force may be in the form of universal law of gravitation. **5**

   b) State Hamilton's least action principle. Derive Hamilton's equations from the variational principle. **10**

OR

4. a) Explain the term Canonical transformation. Obtain simpletic condition for a canonical transformation. **10**

   b) Express the total time derivative $\frac{dA}{dt}$ of a function $A(p_k, q_k, t)$ in terms of the Poisson bracket of A with the Hamiltonian H. **5**

5. a) Describe Hamilton-Jacobi method for solving the equation for a one dimensional linear harmonic oscillator and obtain the solution. **10**

   b) State and prove parallel axis theorem. **5**

OR

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6. a) What are the principal moments of inertia of a rigid body ? Classify rigid bodies based on their principal moments of inertia. **5**

   b) Setup the Euler equations of motion for a rigid body. **10**

7. a) Discuss the simultaneity in relativity. **5**

   b) Obtain the Lorentz covariant form of Newton's second law of motion. Determine the 4th component of the 4-force along the world line of the particle assuming that the rest mass of the particle is finite, real and non-zero it does not vary along the world line. **10**

OR

8. a) Obtain the expression for relativistic energy of a particle of rest mass m. **5**

   b) What is the difference between inertial mass and gravitational mass ? Describe the Eötvös experiment to demonstrate the identity between inertial mass and gravitational mass. **10**

9. Answer **any four** of the following : **(4×5=20)**

   a) Verify whether $\vec{F} = (3x^2 - 3y^2)\,\hat{i} + (4x - 6xy)\,\hat{j}$ is a conservative force or not.

   b) Write down the Lagrange's equations, when the Lagrangian function is

   $L = \dot{q}_k q_k - (1 - \dot{q}_k^2)^{-1/2}$,

   c) One of the attempts at combining the two sets of Hamilton's equations into tries to take q and p as forming a complex quantity. Show directly from Hamilton's equations of motion that for a system of one degree of freedom the transformation $Q = q + ip$, $P = Q^*$ is not canonical if the Hamiltonian is left unaltered. Can you find another set of coordinates Q', P' that are related to Q, P by a change of scale only and that are canonical ?

   d) Using the fundamental Poisson brackets show that the following transformation is canonical

   $Q = \sqrt{2qe^t}\cos p$ and $P = \sqrt{2qe^t}\sin p$.

e) A rigid body in motion has $\vec{\omega} = 2\hat{k}$ and the moment of inertia tensor is

$I = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 6 \end{pmatrix}$. Calculate the angular momentum vector $\vec{L}$ of the rigid body.

f) If T be the kinetic energy, G the external torque about the instantaneous axis and $\vec{\omega}$ the resultant angular velocity, show that $\frac{dT}{dt} = G\vec{\omega}$.

g) At what speed does a matter stick move if its length is observed to shrink to 0.75 m.

h) A body of mass 10 m_e is moving with a velocity $\vec{v} = \frac{c}{\sqrt{2}}\hat{i} + \frac{c}{\sqrt{3}}\hat{j}$, calculate all components of its four momentum.